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ABSTRACT

This study sought to determine whether a number of specific counting and numeration behaviors emerge within children in a fixed developmental sequence; at what point in the development of mathematical behavior the use of numerical representations normally appears; and what relationship holds between development of counting skills and development of one-to-one correspondence operations. The subjects were 78 kindergarten children in an urban public school. They were given a battery of tests, each assessing the ability to perform a specific task involving counting, use of numerals, or comparison of set size. The test scores were subjected to scalogram analyses. Results suggested: (1) a reliable sequence of skills in using numerals; (2) the dependence of learning numerals upon prior acquisition of counting skills for sets of the size represented; (3) acquisition of numeral reading for small sets before learning to count larger sets; and (4) the independence of counting and one-to-one correspondence operations in young children. The implications of these findings for designing an introductory mathematics curriculum are discussed. (Author/CT)

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ABSTRACT

Kindergarten Ss were administered a battery of tests, each assessing ability to perform a specific task involving counting, use of numerals or comparison of set size. The test scores were subjected to scalogram analyses in order to test hypotheses concerning sequences of acquisition of these behaviors. Results suggested: a) a reliable sequence of skills in using numerals; b) the dependence of learning numerals upon prior acquisition of counting skills for sets of the size represented; c) acquisition of numeral reading for small sets before learning to count larger sets; and d) the independence of counting and one-to-one correspondence operations in young children. The implications of these findings for designing an introductory mathematics curriculum are discussed.

THE SEQUENCE OF DEVELOPMENT OF SOME EARLY MATHEMATICS BEHAVIORS

by

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This paper examines the sequence in which young children acquire elementary mathematical behaviors of counting, one-to-one correspondence and numeration. Studies of these very early aspects of a developing concept of number have been relatively rare, with most psychological research on children's mathematical development focussing on the emergence of conservation and related abilities associated with the stage of concrete operations (Piaget, 1965; e.g., Lovell, 1966; Sigel & Hooper, 1968).

Wohlwill (1960) carried out a scalogram analysis of a number of tasks leading to the development of the number concept. Several of his tasks involved matching sets according to the number of objects they contained. However, none required overt counting of objects or observable operations of one-to-one correspondence, and no tasks involving numerals were included. Using a similar methodology, D'Mello and Willemsen (1969) studied four simple number tasks. They established a sequence of rote counting (reciting the numerals in order), followed by matching sets with identical physical arrangements (as in dominoes), counting out a set of specified size and matching a numeral with a set.

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The behaviors involved in the actual act of counting have been experimentally analyzed in studies by Potter and Levy (1968) and Beckwith and Restle (1966), both of which examined the processes involved in keeping track of which members of a set have been enumerated. Potter and Levy showed that the ability to touch each item in a set once and only once increased with age (up to 4 years 3 months, the oldest in the sample) and that arrangement of the objects affected the degree of difficulty of the task. Children able to count objects made fewer errors in enumeration than children who could not successfully count, confirming the authors' hypothesis that skill in enumeration is a key component of rational counting.

Beckwith and Restle used latency as well as error measures in studying the ways in which children and adults organized various arrays of items in order to enumerate in the process of counting. Their results suggest a serial chain of behaviors for counting large sets of objects, in which the objects are first visually grouped into subsets and then successively counted. Children appeared to use different grouping strategies than adults.

Three basic classes of early mathematical behaviors were examined in the present study: a) counting objects, b) using numerals and c) comparison of set size. By examining developmental sequences within and between these classes, the study sought to determine whether a number of specific counting and numeration behaviors emerge in a fixed developmental sequence, at what point in the development of mathematical behavior the use of numeral representations normally appears and what relationship holds between development of counting skills and development of

one-to-one correspondence operations.

Specific hypotheses concerning the sequence of acquisition of the behaviors under study were derived from a process of behavior analysis described by Resnick (1967). On the basis of these analyses, the tasks under study were ordered into hierarchies of successively more complex learning tasks, with skill in performing tasks lower in a hierarchical sequence hypothesized to be prerequisite to learning those higher in the same sequence (cf. Gagne, 1962). The predicted hierarchies are shown in Figures 1 and 2. In each figure the tasks hypothesized to be the most complex appear at the top, with prerequisites shown below, connected by a line. In each box in these figures, the entry above the line describes the stimulus situation, and the entry below the line describes the appropriate response.

Figure 1 shows a hierarchy of tasks concerned with counting and using numerals. A sequence of counting tasks is defined by cells D-E-F-H-I. A similar sequence of tasks for use of numerals is given in cells A-B-C. Tasks G1 and G2 combine the counting and numeration skills. The placement of these tasks in the middle of the hierarchy reflects a hypothesis that numeration is normally learned early in the process of learning to count. The separate lines for A-B-C and D-E-F reflect an initial hypothesis that counting and numeration skills are learned independently, with neither class of tasks prerequisite to the other. Although not shown in the diagram, it was further hypothesized that both counting and numeration skills for quantities up to five would normally be learned before either class of skills was learned for quantities up to ten.

Figure 2 shows a hierarchy of tasks concerned with comparing sets, either through a process of one-to-one correspondence (cells A-B-C) or by counting (cells D-E-F). In mathematical theory, one-to-one correspondence occupies a central position as the basis of the concept of number. Most modern mathematics teaching, therefore, places heavy initial emphasis on operations leading to the concept of one-to-one correspondence of sets; counting operations are treated as deriving from one-to-one correspondence. Behavior analyses of one-to-one correspondence tasks and counting tasks, however, revealed virtually no common components. Thus, from a behavioral point of view there was no reason to treat either type of task as prerequisite to the other. The hypothesis of psychological independence of counting and one-to-one correspondence skills is reflected in the two independent sequences shown in Figure 2.

Method

Subjects

Ss were 78 kindergarten children in four classes in an urban public school. Ss ranged from 4 years 6 months to 6 years 0 months, with a median of 5 years 4 months. Forty-two of the children were boys and 36 were girls. Sixty-three percent of the Ss were Black, 37 percent White. Occupations of parents in the school ranged from unemployed through executive-professional level; however, the distribution was skewed in the direction of lower socioeconomic status: 22.9 percent of student families had no father at home; of fathers at home, the median occupation was that of a semi-skilled laborer. Children were assigned randomly to the classes in the school, and all children in each class were included in the basic sample. Thus, the socioeconomic characteristics of the sample used in this study

should closely match those of the school as a whole.

Tests

A test was prepared for each behavior specified in the hypothesized hierarchies (Wang, 1968). These tests were designed for oral, individual administration. Each test consisted of one to five items. All tests were scored dichotomously. A subject was rated as passing a test only if he passed all items in the test; otherwise, he was scored as failing. For each of the tasks described in Figure 1 (labelled 1:A through 1:I below), two separate tests were given, one covering sets or numerals from zero to five, and one covering sets or numerals from six to ten. For each of the tasks described in Figure 2 (labelled 2:A through 2:F), there was a single test using set sizes up to ten.

1:A (match two sets of numerals). A set of cards, each with a numeral written on it, was arranged in a row (but not in numerical order) on the table. The child was handed a card from a second, matching, set and asked to, "Find the one on the table that is the same." The process was repeated for all numerals in the set. If the child picked up a card from the table, it was replaced in position before proceeding to the next item.

1:B (select a stated numeral). A set of cards, each with a numeral written on it, was arranged on the table. _E_ said, "Hand me the numeral (two)." The procedure was repeated for all numerals in the set in a preselected random order. _E_ replaced the card the child handed him before asking for the next numeral.

1:C (read numerals). The child was shown a card with a numeral written on it and asked, "What numeral is this?" The procedure was repeated for all numerals in a preselected random order.

1:D (recite numerals in order). The experimenter said, "Count to five (ten)." If the child did not respond, E prompted him by saying "One... two..." The child was then asked to count "by himself."

1:E (count moveable objects). For the 0-5 test, 2 inch-square colored "counting cubes" were placed randomly on the table. E said, "How many objects are there? Count them." The process was repeated using 5 cubes, then 4 cubes. For the 6-10 test, the three items used 6, 8 and 10 cubes.

1:F (count out subset of objects). For the 0-5 test, 10 counting cubes were placed randomly on the table. E said, "Count out 3 objects from this pile and put them over here (pointing to another place on the table)." The process was repeated with E requesting 5, and then 2 objects. For the 6-10 test, E requested 7, 10 and 8 objects from an initial set of 15. In both tests, E replaced the blocks the child had counted out before making a new request.

1:G1 (count out subset of size indicated by a numeral). For the 0-5 test, 10 counting cubes were placed in a pile on the table. E handed the child a card with the numeral 3 on it and said, "Read the numeral. Put as many objects as the numeral says below the card." This was repeated for the numerals 1 and 5. For the 6-10 test, 25 objects were placed on the table, and cards with the numerals 9, 6 and 8 were used.

1:G2 (match a set of objects with the appropriate numeral). E placed a set of cubes on the table and displayed a set of cards, each showing a single

numeral. E said, "Count the objects in this pile. Show me the numeral card that shows the right number." For the 0-5 test, sets of 3, 2 and 5 cubes were used. For the 6-10 test, sets of 8, 7 and 10 were used.

1:H (count ordered array of fixed objects). A sheet of paper showing a row of dots was shown to the child. E said, "How many dots are there? Count the dots." The 0-5 test contained items showing 5, 4 and 1 dots. The 6-10 test used 6, 10 and 7 dots.

1:I (count unordered array of fixed objects). A sheet of paper showing dots randomly organized was shown to the child. E said, "How many things are there on this card? Count them." The 0-5 test showed 2, 5, 4, 1 and 3 dots. The 6-10 test showed 6, 8, 10, 9 and 7 dots.

2:A (pair two sets, state whether equal). Two sets of 6 cubes each were placed in separate piles on the table. E asked the child to line up one set, then to line up the second set in a paired relationship with the first. E demonstrated pairing, if necessary. When the objects were arranged, E asked, "Do these two rows have an equal number of objects?" E then placed a pile of 8 cubes and a pile of 4 cubes on the table, asked S to arrange them and then asked, "Do these two rows have an equal number of objects?" No demonstration or prompting was given on the second item.

2:B (pair two sets, state which has more/less). The procedure was identical to the one used in test 2:A, except that when the objects were lined up E asked, for the first item (sets of 6 and 4 cubes), "Which line has more objects?" and for the second item (sets of 5 and 3 cubes), "Which line has less objects?"

2:C (pair three sets, state which has most/least). Three piles of cubes (5, 8 and 2) were placed on the table, and the child directed to line up each pile, establishing one-to-one correspondence between the sets. He was then asked which row (line) had the most objects. For the second item, 4 piles (5, 6, 4 and 9 cubes) were used, and S was asked to point out the line with the least objects.

2:D (count two sets, state whether equal). Two sets of cubes were placed in separate piles on the table and E said, "Count the objects in each of these piles. Do the two piles have the same number of objects?" There were two items in the test, the first comparing sets of 7 and 6 cubes, the second comparing two sets of 7 cubes each.

2:E (count two sets, state which has more/less). The procedure was identical to task 2:D except that E asked for the first item (8 vs. 5 cubes), "Which pile has less?" and for the second item (6 vs. 3 cubes), "Which pile has more?"

2:F (count three sets, state which has most/least). Three sets of cubes were placed in separate piles on the table. E asked the child to count the objects in each pile and to say which pile had the most (then the least). There were two items, the first comparing sets of size 7, 4 and 6; the second comparing sets of size 10, 5 and 8.

Procedure

Six trained research assistants served as E's--Ss were assigned to them in random order. In order to minimize learning effects, four different

sequential orders for testing were specified. All 78 Ss were tested on each of tests 1:A through 1:I. Thirty-seven of these Ss were also given tests 2:A through 2:F. Ss were tested individually in a special testing area outside the classroom. Each testing session lasted approximately 20 minutes. The number of sessions required for individual Ss ranged from one to three. The total testing period was about four weeks. The testing began on the third week of school and teachers agreed not to engage in any mathematics teaching until the testing for this study was completed. Since Ss were kindergarteners, with little or no prior formal schooling, the results to be reported thus constitute an assessment of relatively "natural" sequences of acquisition for children in the American urban environment, rather than of sequences imposed in the course of formal mathematics instruction.

Methods of data analysis

If the hypothesized hierarchical relationships among objectives are correct, then for any given linear pathway (e.g., Figure 1:A-B-C-G-H-I), a subject who passes a test should also pass all tests lower in the sequence. Conversely, working up the sequence, once a subject fails a test, he should fail all succeeding tests. The tests, in other words, should form a Guttman scale (Guttman, 1944).

In the present study, Lingoes' (1963) method of Multiple Scalogram Analysis (MSA) was used to test the scalability of subsets of tests. Rather than simply accepting or rejecting a hypothesized sequence, MSA reorders the tests in the set so as to suggest an optimal scaling sequence. Where all of the objectives

cannot be ordered in a single scale, MSA will suggest two or more independent scales. A test is accepted for membership in a scale if it meets a preset criterion of reproducibility with all tests already accepted. 1.00 indicates perfect reproducibility--i.e., the criteria for a Guttman scale, as given above, are met with no exceptions. For the present studies, the minimum reproducibility criterion was set at .80. MSA controls statistically for spuriously high estimates of reproducibility due to extreme pass or fail rates, a problem which has engendered criticism of Guttman scaling methods in the past (Festinger, 1949; Green, 1956).

Although the MSA program is capable of picking out multiple scales, these scales are independent of one another, having no objectives in common. Once an objective is selected for inclusion in a scale, it is no longer considered for membership in other scales. For example, with respect to Figure 1, if objective G were to scale with C, B and A, it could not, in the same analysis, appear in a scale with F, E and D. Therefore, it was necessary to test separately each of the linear pathways implied by the hierarchies under study.

Results

1. Scalability Within Classes of Behaviors

Separate analyses for the three basic classes of behaviors under study were undertaken first. These analyses covered: a) counting objects, b) using numerals and c) comparison of set size.

a) Counting objects. Figure 1 includes seven behaviors which involve counting of objects, in the hypothesized sequence D-E-F-G1-G2-H-I.

The predicted hierarchy was tested separately for counting from 0 to 5 objects and for counting from 6 to 10 objects. Table 1 shows both the hypothesized scale (left-hand column) and the empirical scales generated by MSA (columns 2 and 4), together with the percentage of correct responses on each test (columns 3 and 5). Both the 0-5 and the 6-10 tests yielded linear scales, but with considerable deviation from the predicted sequence. In both cases, tasks G1 and G2, which involve the use of numerals in conjunction with counting, appear as the highest order behaviors rather than occupying the predicted middle position in the hierarchy. In other words, virtually all Ss who were able to associate numerals with sets were also able to perform all of the object counting tasks specified in E, F, H and I. This finding suggests that numerals are not ordinarily learned until counting is a well established skill, an implication that will be examined more directly later in this paper.

A reordering of the counting skills themselves is also suggested by these data. Counting out a subset of specified size (task F) is apparently learned later than counting a given set (tasks E, H and I), regardless of how the set is presented. For Ss able to count at all, the most typical error on F was to continue counting out objects beyond the number specified in the instruction. The Ss, in other words, could count objects adequately, but could not remember the number requested as they counted. Thus, the addition of a "memory component" is very likely the factor that places task F near the top of the hierarchy.

Tasks E, H and I appear in two different orders for 0-5 and 6-10. This discrepancy can probably be explained by the nature of the physical arrangement of the objects to be counted. In H, the objects are pictures presented in one or two straight rows. In I, the pictures are scattered in random fashion, presumably imposing on the child the additional burden of visually "ordering" the objects in order to keep track of which ones have already been counted. However, when only five or fewer objects are involved, almost any arrangement will look "ordered." Thus, for 0-5 objects, the data show virtually no difference in difficulty between H and I. For six or more objects, on the other hand, a randomly arranged set can pose real difficulty in ordering as is reflected in the sharply lowered pass rate for I.

Task E requires the counting of objects rather than pictures. These objects can be removed from the set as they are counted, thus providing a particularly easy means of keeping track of which objects have been counted. For this reason, E was placed near the bottom of the predicted hierarchy. In practice, most Ss failed to take advantage of the possibility of removing already counted objects and instead treated the set as if the objects were fixed in place. Since the tester placed the objects randomly on the table, the test thus became most similar to that for task I (counting unordered arrays). This similarity is particularly reflected in the data for the 6-10 tests. These data suggest that, in the absence of explicit instruction, counting is not necessarily learned as a process of successive removal of objects in the set.

b) Using numerals. Figure 1 shows five behavior involving numerals, in the order A-B-C-G1-G2. Results of MSA analysis for 0-5 and 6-10 appear in Table 2. The predicted sequence was verified for both levels. The sequence begins with perceptual matching of numerical forms (A), proceeds through recognition (B) and naming (C) numerals and concludes with the association of sets and numerals (G).

c) Comparison of sets. Figure 2 shows two independent sequences of behaviors involving set comparison: A-B-C and D-E-F. Table 3 shows the results of MSA analysis for the two independent sequences. Task F was excluded from the analysis because no Ss passed it. Only 5.4 percent of the Ss passed test C and this test did not scale with A and B. These low rates of passing suggest that comparisons of three or more sets involve behavioral components (such as skill in arranging objects, retaining numbers in memory, etc.) that go considerably beyond the mathematical concepts involved in comparing two sets.

The relative difficulty of making "same-different" (tasks A and D) as opposed to "more-less" (tasks B and E) judgments with respect to quantity is unclear from these data. The predicted sequence ("same" prerequisite to "more" and "less") was confirmed for the one-to-one correspondence comparisons, but reversed for the counting comparisons. Other research on these concepts, using various testing conditions, has yielded conflicting results (cf. Donaldson & Balfour, 1968; Uprichard, 1970). Further research based on careful analysis of the terms and behaviors required in various tests is clearly needed.

2. Relations Between Classes of Behaviors

Having established the existence of some regular sequences of acquisition within limited classes of behaviors, it is of interest to examine how the different classes of behaviors are related to each other. Scalogram analysis for combined subsets of objectives can provide information concerning such relationships. Several such analyses were performed in the present study.

a) Combined scale for counting objects and using numerals. Analysis 1a (see Table 1) showed tasks G1 and G2, which involve the association of numerals with sets, at the very top of an empirical hierarchy of counting tasks. This finding was interpreted as suggesting that knowing how to count objects is prerequisite to learning numerals. A more direct test of this interpretation can be made by applying MSA to the combined set of counting (1:D, E, F, H, I) and numeral (1:A, B, C) test scores. The analysis was performed separately for sets and numerals to 5 and sets and numerals to 10. Results appear in Table 4. Two predicted scales are shown, one derived from the original hierarchy shown in Figure 1, the second (the "revised" column) based on the results of analysis 1a.

For 0-5 and 6-10, task A, visual matching of numeral shapes, appears first in the hierarchy, indicating it is learned even before rote counting. Task A, however, is a purely visual task: any shapes might be matched. When numeral names are required, either as stimulus (task B) or response (task C), the task is considerably more difficult. These tasks, in fact, appear at the top of the empirical hierarchy. With a single exception (task F for 0-5), all counting tasks

are prerequisite to learning numeral names. The interpretation offered for analysis 1a is thus confirmed. This finding implies that while a child might theoretically be taught to recognize and read numerals as a kind of rote paired-associate task, he would probably learn more easily after considerable experience with counting objects.

b) Combined scale for counting and set comparison. It was initially hypothesized that set comparison by one-to-one correspondence (Figure 2: tasks A, B, C) was neither prerequisite to nor dependent on counting (Figure 1: tasks D, E, F, H, I). Furthermore, set comparison by counting (Figure 2: tasks D, E, F) was initially hypothesized as independent of one-to-one correspondence, but dependent on counting skills. The predicted relationship among the classes of behaviors is shown graphically in Figure 3.

In order to test the relations between the four classes of tasks shown in Figure 3, the tasks in each class were treated as a group and given a single pass or fail score. Ss were scored as passing Counting Objects to Five (Class I) if they passed three of the five counting tests, 1:D, E, F, H, I, for 0-5 objects. They were similarly scored as passing Counting Objects to Ten (Class II) if they passed three of the five counting tests for 6-10 objects. Passing scores for Comparison of Sets by Counting (Class III) were given for passing either 2:D or 2:E. Ss were counted as passing Comparison of Sets by One-to-One Correspondence (Class IV) if they passed either 2:A or 2:B.

MSA was run for the Class I-II-III sequence. Results, shown in Table 5, suggest that while the behaviors do form a single scale, comparison behaviors

occur before children are able to count large sets of objects. To test the independence of counting and one-to-one correspondence operations, the pass-fail relationships for Class IV and each of the other three classes were then examined. The relevant contingency tables appear in Table 6. No association or dependency relationship between Class IV and the other three classes is suggested by these data.

c) Combined scales for counting and numerals to five and counting and numerals to ten. The division of counting and numeral skills into two levels provided the opportunity to make two independent tests of certain sequences of behaviors. The division was made on the basis of informal observations suggesting that children learn both counting and numerals for the smaller quantities substantially sooner than for higher quantities. To empirically verify this observation, a scalogram analysis of four classes of behaviors was conducted. The classes were: Counting Objects to Five (Class I); Counting Objects to Ten (Class II); Using Numerals to Five (Class V) and Using Numerals to Ten (Class VI).

For Classes I and II, pass scores were assigned for passing three of the five counting tests. For Classes V and VI, a pass score was assigned to Ss who passed test 1:C (reading numerals). Table 7 shows the MSA results for the four classes of tasks. The four classes form a single scale in which counting and numeral tasks for quantities up to five were prerequisite to both types of task for quantities to ten. This finding confirms the initial hypothesis.

Discussion

The data reported here suggest strongly the existence of several reliable sequences of development of mathematical behavior in young children.

Specifically, the data suggest that: a) command over numerals is acquired in a regular sequence beginning with perceptual matching of the numerals and concluding with the association of sets and numerals. This sequence was predicted on the basis of behavior analysis and was confirmed independently for numerals 0-5 and 6-10. It thus can be considered a relatively firm conclusion, at least for the type of population studied. b) Numerals are learned only after counting operations for sets of the size represented by the numerals are well established. This sequence was not predicted, but was established post-hoc, and thus requires confirmation using a new sample of Ss. c) Counting and numeration for small sets (up to five) are acquired before counting larger sets is learned.

The data were unclear with respect to the order with which specific types of counting behaviors are acquired and with respect to the relationship between the concepts "same," "more" and "less." Thus, with the exception of the sequence for numeral use, the study offers greater clarity with respect to the developmental relationship between classes of mathematical behavior than with respect to details of acquisition sequence within classes.

In addition to confirming predicted sequences of behavior acquisition, the study also offers support for a predicted independence of two classes of mathematical behavior, counting and one-to-one correspondence. The mathematical definition of number is based on the one-to-one correspondence properties of sets, and counting is, therefore, often treated as a derivative of one-to-one correspondence in mathematical thought. However, analyses of the actual behaviors involved in counting sets and in establishing correspondences between them suggested that the two classes of behavior should be psychologically

independent with respect to sequence of acquisition. This hypothesis was supported by the data.

Since the Ss in the experiment had not yet been exposed to formal instruction in mathematics, the scales established in this study reflect sequences of acquisition that are "natural," at least in the community studied, rather than artificial sequences imposed in the course of schooling. The existence of such sequences, particularly if they can be replicated in other cultural contexts, suggests that school curricula might be developed which, by paralleling the sequence of development usually found outside of formal instruction, could be expected to optimize ease and speed of school learning. On the basis of present findings, for example, an optimal introductory mathematics curriculum might be expected to stress counting operations, introducing them simultaneously with one-to-one correspondence. Such a curriculum would delay introduction of numerals until counting was well established, but it would probably introduce numerals for small sets as soon as the child could count those sets, rather than waiting until more extended counting skills had been developed.

With respect to such applications, however, it is important to note that sequences of behavior based on scaling data only suggest, but do not directly confirm, hypotheses concerning instructional efficiency. Where specific sequences are to be prescribed in instruction, the sequences should be those that will provide maximum transfer from earlier to later learned behaviors. Such transfer relationships can be directly tested only in

experimental designs involving instruction or practice in the various behaviors in a hypothesized sequence (cf. Resnick, 1970). A body of such research, combined with studies of the present type, would permit empirical examination of the correlation between natural sequences of acquisition in a given culture and optimal instructional sequences.

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TABLE 1

Multiple Scalogram Analysis of Behaviors Involving

Counting Objects

N=78

Hypothesized Scale	0-5		6-10	
	Empirical Scale	Percent Correct Responses	Empirical Scale	Percent Correct Responses
I	G2	30.8	G2	19.2
H	G1	37.2	G1	20.5
G2	F	50.0	F	23.1
G1	H	59.0	I	24.4
F	I	59.2	E	26.9
E	E	66.7	H	43.6
D	D	87.2	D	67.9
Reproducibility	.963		.923	

TABLE 2

Multiple Scalogram Analysis of Behaviors Involving

Numerals

N=78

Hypothesized Scale	0–5 Empirical Scale	0–5 Percent Correct Responses	6–10 Empirical Scale	6–10 Percent Correct Responses
G2	G2	30.8	G2	19.2
G1	G1	37.2	G1	20.5
C	C	42.3	C	21.8
B	B	51.3	B	21.8
A	A	91.0	A	78.2
Reproducibility	.987		.985	

TABLE 3

Multiple Scalogram Analysis of Behaviors Involving

Comparison of Set Size

N=37

Hypothesized Scale	Empirical Scale 1	Percent Correct Responses	Empirical Scale 2	Percent Correct Responses
<u>One-to-one correspondence:</u>				
C	B	24.3	C	5.4
B	A	32.4		
A				
Reproducibility	.932		1.000	
<u>Counting:</u>				
E	D	24.3		
D	E	27.0		
Reproducibility	.905			

TABLE 4

Combined Analysis of Counting Objects and Using Numerals

N=78

Hypothesized Scales: Original	Hypothesized Scales: Revised	0-5: Empirical Scale	0-5: Percent Correct Responses	6-10: Empirical Scale	6-10: Percent Correct Responses
K^c	C^n	C^n	42.3	C^n	21.8
H^c	B^n	F^c	50.0	B^n	21.8
C^n	A^n	B^n	51.3	F^c	23.1
B^n	F^c	H^c	59.0	I^c	23.1
A^n	I^c	I^c	59.2	E^c	26.9
F^c	H^c	E^c	66.7	H^c	43.6
E^c	E^c	D^c	87.2	D^c	67.9
D^c	D^c	A^n	91.0	A^n	78.2
Reproducibility		.932		.921	

^{n} Numeral Task ^{c} Counting Task

TABLE 5

Multiple Scalogram Analysis of Counting and Set Comparison by Counting

N=37

Hypothesized Scale	Empirical Scale	Percent Correct Responses
Class III (Comparison by Counting)	II	37.8
Class II (Counting objects to 10)	III	45.9
Class I (Counting objects to 5)	I	59.5
Reproducibility	.928	

TABLE 6

Pass-Fail Contingencies for Counting and One-to-One Correspondence Behaviors

		Class I P	Class I F
Class IV	P	15	3
	F	9	10

(a)

		Class II P	Class II F
Class IV	P	10	8
	F	4	15

(b)

		Class III P	Class III F
Class IV	P	11	7
	F	6	13

(c)

TABLE 7

Multiple Scalogram Analysis of Classes of

Counting and Numeral Behaviors

N=37

Empirical Scale	Percent Correct Responses
Class VI (Numerals to 10)	27.0
Class II (Counting objects to 10)	37.8
Class V (Numerals to 5)	40.5
Class I (Counting objects to 5)	59.5
Reproducibility .953	

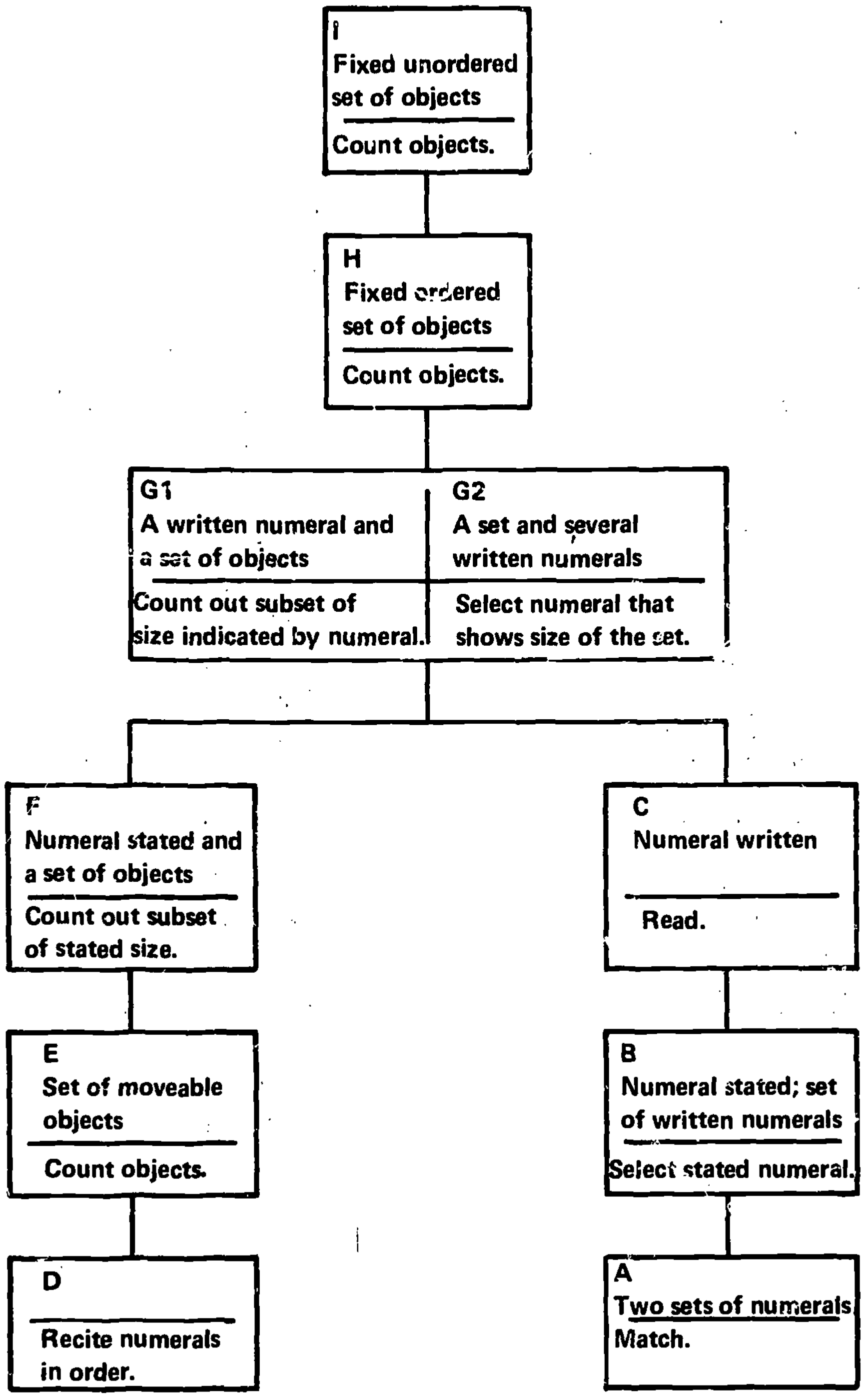

Figure 1 : Hypothesized Hierarchy of Counting and Numeration Tasks.

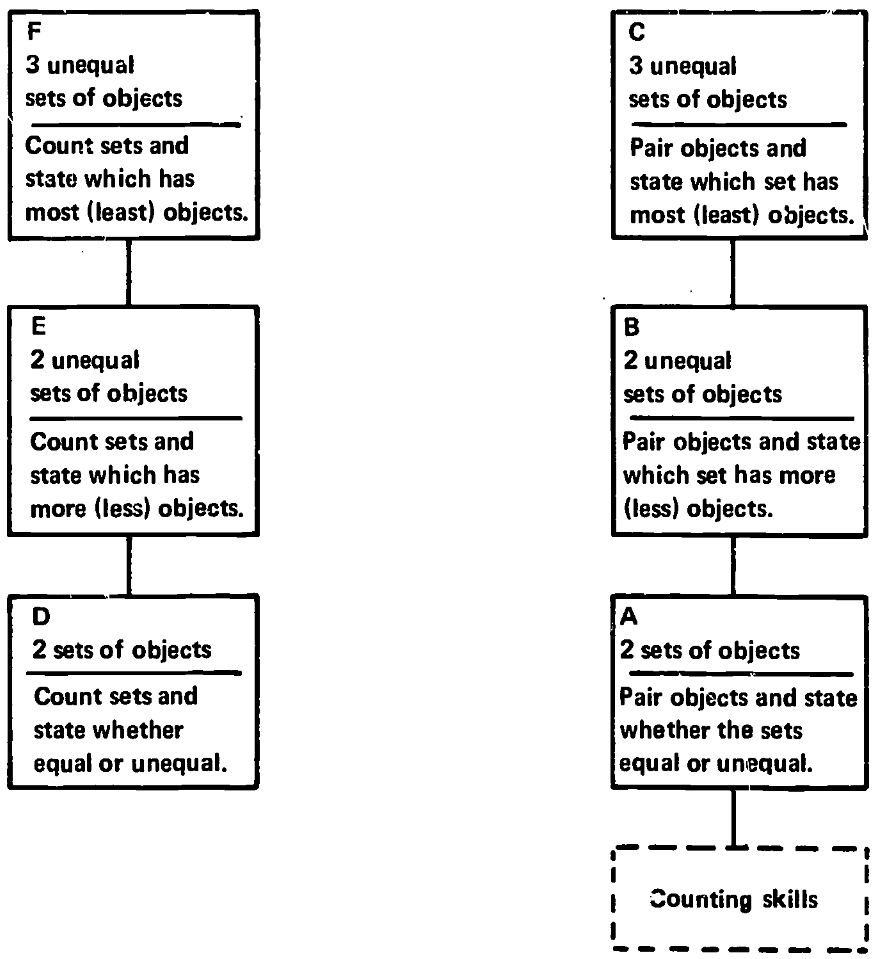

Figure 2: Hypothesized Hierarchy of Set Comparison Tasks.

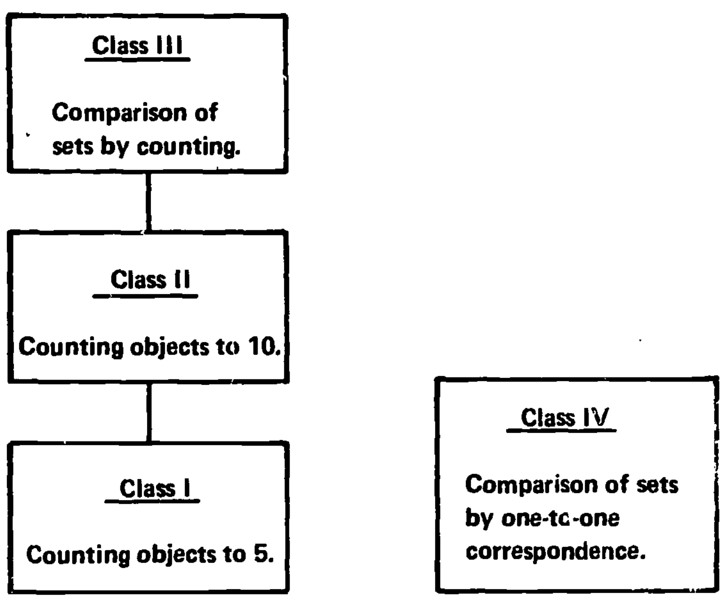

Figure 3: Hypothesized Hierarchy of Classes of Counting and Set Comparison Tasks.